

# Composite Sandwich Structure Optimization with Application to Satellite Components

Srinivas Kodiyalam\* and Somanath Nagendra†  
General Electric Corporate R&D Center, Schenectady, New York 12301  
and  
Joel DeStefano‡  
Lockheed Martin Astro Space, Princeton, New Jersey 08543

Advanced design concepts and rigorous optimization methods are essential to address the conflicting and stringent design requirements of aerospace structures. The focus here is on the use of discrete optimization methods, specifically the genetic search method, for tailoring composite sandwich components of satellites. Failure modes induced by local instabilities associated with such composite sandwich constructions have been developed and used as design constraints in the optimization problem. A linear least-squares approximation is used in conjunction with the genetic search procedure to minimize the computational effort. Results of this study indicate that, for large-scale design optimization problems, the number of function evaluations required by this implementation is high and that more robust and alternate approximation concepts are necessary to minimize the overall computational effort.

## Nomenclature

$C_1, C_2$	= wrinkling engineering constants
$E_c$	= Young's modulus of core
$E_x, E_y$	= Young's modulus skin (x and y directions in global coordinate system)
$F(X)$	= design optimization objective function
$F_b$	= adhesive bonding strength
$F_c$	= flatwise sandwich strength
$F_{cx}, F_{cy}$	= compressive strength allowables (x and y directions in global coordinate system)
$F_{cxy}$	= in-plane shear strength allowable
$F_{tx}, F_{ty}$	= tensile strength allowables (x and y directions in global coordinate system)
$f(N)$	= least-squares function approximation
$G_{cx}, G_{cy}$	= core shear modulus (x and y directions in global coordinate system)
$g_j(X)$	= jth design constraint
$g_{lb}$	= lower bound on design constraint
$g_{MS}$	= margin of safety design constraint
$g_{ub}$	= upper bound on design constraint
$g_\omega$	= frequency constraint
$S$	= population size
$S_{11}, S_{22}, S_{12}$	= in-plane ply stresses in fiber and matrix directions (material coordinate system)
$S_{13}, S_{23}$	= interlaminar shear stresses (material coordinate system)
$s$	= honeycomb-core cell size
$t_c$	= honeycomb-core thickness (height of core)
$t_f$	= composite faceskin thickness
$t_w$	= honeycomb-core wall thickness
$W$	= structural weight
$w$	= surface waviness
$X$	= vector of design variables
$x_i$	= ith design variable

$\beta, \gamma$	= penalty parameters for augmented objective function
$\mu_x, \mu_y$	= Poisson's ratio (x and y directions in global coordinate system)
$\sigma_{allow}$	= minimum allowable stress
$\sigma_{max}$	= maximum stress
$\phi$	= augmented objective function or fitness function

## I. Introduction

THE need to reduce weight, cost, and design cycle time while improving product performance and reliability are primary drivers in the design of satellites. The use of advanced design concepts and rigorous optimization methods, therefore, becomes essential to seriously address the conflicting and stringent design requirements and thereby improve the overall quality and reliability of satellite structural designs.

Composite materials and honeycomb sandwich structures that can allow for significant weight savings and provide for high stiffness-to-weight ratios are seeing an increased rate of infusion into satellite structures. Such materials and constructions are now being used for primary and secondary structures and for structures intrinsic to subsystems such as solar arrays and reflectors. Great care must be exercised in the design optimization of these structures and components, however, because of the several failure modes possible with sandwich constructions when it is subject to a wide range of loading conditions.<sup>1</sup>

Some of the earlier works in design optimization of composite sandwich structures, specifically sandwich panels, are reported in Refs. 2 and 3. In Ref. 2, analytical procedures are developed to provide a means of selecting unique stacking sequences for composite faceskins of sandwich panels, subject to various in-plane loads to produce minimum weight panels. Numerical optimization techniques<sup>5</sup> are used in Refs. 3 and 4 for truss-core sandwich structural designs and stiffened composite panel components. Sandwich constructions, compared to stiffened skin constructions, provide for inherent stability against buckling and eliminate the need for stringers.

This paper focuses on the use of discrete optimization methods, specifically the genetic search method, for tailoring composite sandwich structures and components of satellites, subjected to transient, acoustic pressure and on-orbit thermal soak loading conditions. Genetic algorithms (GAs) are emerging as a viable tool for dealing with the problem of discrete design variables and provide the designer with multiple optima as against a single optimum solution.<sup>6</sup> The use of GAs as an approach to solving nonconvex engineering

Received March 13, 1995; revision received Oct. 8, 1995; accepted for publication Oct. 15, 1995. Copyright © 1995 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

\*Staff Engineer, Engineering Mechanics Laboratory, Building K1, Room 2A25, P.O. Box 8; currently Senior Staff Scientist, Structures Laboratory, Research and Development Division, Lockheed Martin Corporation, Palo Alto, CA 94304-1191. Senior Member AIAA.

†Staff Engineer, Engineering Mechanics Laboratory, Building K1, Room 2A25, P.O. Box 8. Member AIAA.

‡Staff Engineer, Spacecraft Mechanical Analysis Group.

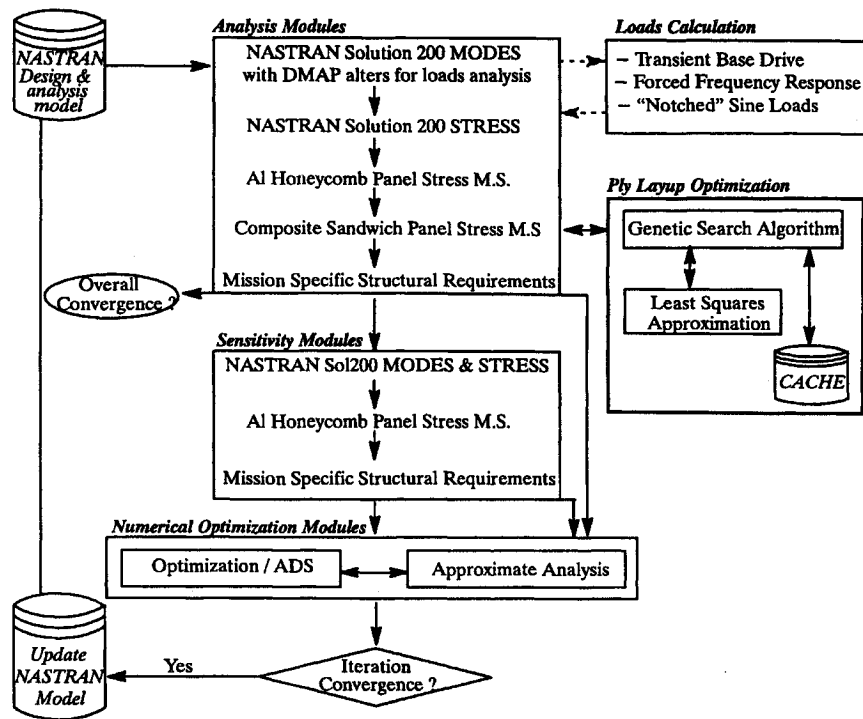


Fig. 1 SAT-OPT system architecture.

and composite structural design problems is reported in Refs. 7–9. In Ref. 10, the laminate stacking sequence design problem is solved subject to buckling and strength constraints. In Refs. 11 and 12, the same approach is applied to the design of unstiffened composite panels and stiffened composite panels.

Despite the successful use of genetic search algorithms in the ply lay-up optimization of composites, a major drawback of this search technique is that it often requires a high number of function evaluations and therefore requires a prohibitively high amount of computational time for large-scale, finite element-based structural optimization problems. This drawback is offset to some extent by the fact that GAs provide the designer with multiple local optimal solutions. Approximation concepts have been commonly used with gradient-based structural optimization procedures to minimize the number of complete finite element analyses and thereby reduce the computational time.<sup>13</sup> The work reported in Refs. 14 and 15 addresses the issue of reducing the computational cost of genetic search through the use of certain local least-squares approximations.

In this paper, three satellite component design problems are optimized for laminate lay-up and honeycomb-core parameters using the present implementation of GAs. Both mechanical and thermal design requirements are considered. A linear least-squares approximation with respect to design variables or a distance measure is used for the design objective and constraint functions to minimize the computational time. A new operator, switching, is introduced in conjunction with stacking sequence optimization. The optimization methodology, sandwich structure local instability relations, and results from the design examples are detailed in the following sections.

## II. Satellite Structural Optimization (SAT-OPT) Methodology

A design optimization system for satellite structures, SAT-OPT, is currently being developed at General Electric Corporate R&D Center. SAT-OPT uses rigorous optimization techniques for the design of satellite subsystems and components with the primary objective of improving the overall quality of designs and reducing the engineering design cycle time required to produce optimal designs.<sup>16</sup> SAT-OPT leverages the design model capabilities that currently exist within MSC/NASTRAN (Solution Sequence 200) and exploits the notion of approximate analysis, along with semianalytical design sensitivity calculations, to achieve substantial computational efficiency. Numerical optimization techniques<sup>5,17</sup> are used for sizing

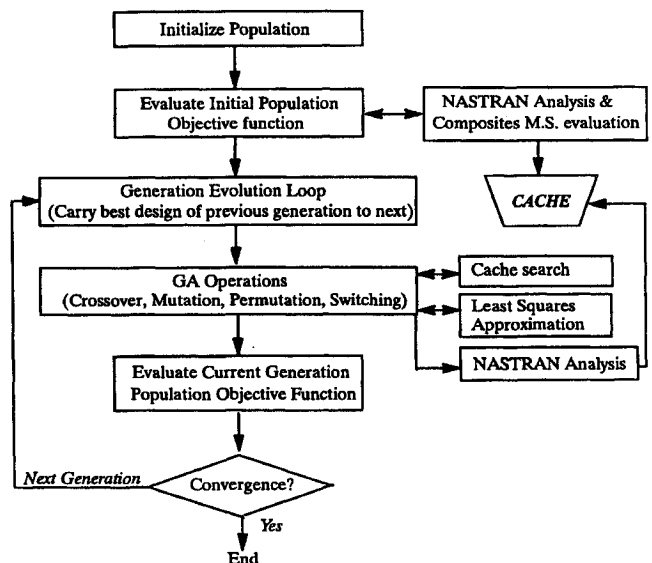


Fig. 2 Genetic search algorithm-based ply lay-up optimization.

the primary and secondary structures (payload panels), as well as the components and subsystems, such as solar arrays and reflectors. Transient flight loads used to size the secondary structures, solar arrays, and reflectors are estimated using a transient base excitation method.<sup>18</sup> A fixed-interface Craig–Bampton model representation of the satellite is used with this base excitation analysis. Response accelerations obtained from transient integration are used to develop actual design loads for secondary structures and subsystems. A flowchart of the satellite structural design optimization methodology is shown in Fig. 1.<sup>16</sup> The details of the GA implementation in SAT-OPT, shown in Fig. 2, are provided in Sec. V.E.

### A. Composite Ply Strength and Sandwich Local Instability Margins of Safety

A composite structure is typically a layered construction composed of a number of plies, each having an initial thickness and orientation angle. In satellite structural constructions, there typically will be several thin plies (facesheets) bonded to a thick honeycomb

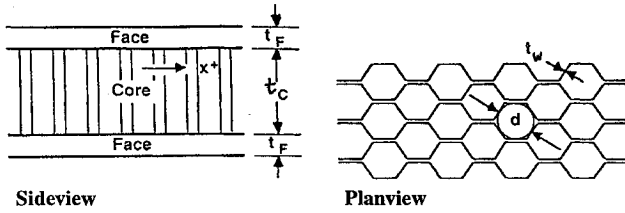


Fig. 3 Honeycomb core.

core (see Fig. 3). The facesheets take the membrane and bending loads while the core resists the shear loads. Based on the type of design load conditions, a composite sandwich structure needs to be designed to meet several different strength and local buckling requirements,<sup>1</sup> such as:

- 1) Facesheets must be sufficiently thick to withstand the tensile, compressive, and shear stresses under design load conditions.
- 2) Core must be designed with sufficient thickness and shear modulus to withstand the shear stresses due to loads and prevent overall buckling, excessive deflection, or shear failure of the facesheet.
- 3) Core cell size and density must be such that the facesheets should not buckle into the core.
- 4) Compressive strength of the core and flatwise tensile strength of the facesheet must be adequate to prevent wrinkling and crimping of the facesheet.

For composite sandwich structure optimization, the listed ply and sandwich strength and local buckling requirements are imposed as constraints on the different margins of safety (MS) in the form

$$g_{MS}(X) = g_{lb} < MS < g_{ub} \quad (1)$$

$$MS = \frac{\sigma_{allow}}{(\text{Factor of safety})\sigma_{max}} - 1.0$$

The Tsai–Wu criterion<sup>19</sup> is used for ply failure margin of safety, and three local buckling criteria, including intercell compression buckling, facesheet wrinkling, and shear crimping, are used for sandwich structure buckling margin of safety.

#### Intercell Compression Buckling

Intercell compression buckling is a local instability failure of the composite facesheets under compressive loads. Thin facesheets can fail locally from buckling under in-plane compression or when insufficient support is provided by the core because of large cell sizes. The empirical equations used for calculating intercell compression buckling strength allowable are functions of facesheet thickness and core cell size and can be evaluated using

$$F_{cx} = \frac{2E_x(t_f/s)^2}{1 - \mu_x^2} \quad (2)$$

$$F_{cy} = \frac{2E_y(t_f/s)^2}{1 - \mu_y^2} \quad (3)$$

$$F_{cxy} = \frac{2(E_x E_y)^{0.5}(t_f/s)^2}{1 - \mu_x \mu_y} \quad (4)$$

#### Facesheet Wrinkling

Wrinkling is a phenomenon in which a facesheet buckles inward or outward, depending on whether there is core compression failure or adhesive bonding failure. Empirical equations for compressive strength used for the facesheet wrinkling strength evaluation are given as

$$F_{cx} = \left( \frac{C_1}{1 + C_2 a} \right) \left( E_c \frac{t_f}{E_x t_c} \right)^{0.5} E_x \quad (5)$$

$$F_{cy} = \left( \frac{C_1}{1 + C_2 a} \right) \left( E_c \frac{t_f}{E_y t_c} \right)^{0.5} E_y \quad (6)$$

$$a = (w E_c)/(t_c F_c) \quad (7)$$

#### Shear Crimping

Shear crimping is a form of local instability caused by small buckling wavelengths of the facesheets leading to the formation of a crimp. The core fails in shear because of low core shear modulus at the crimp. Empirical equations used for calculating the shear crimping strength allowable are given by

$$F_{cx} = \frac{G_{cx}(t_c + 2t_f)}{2t_f} \quad (8)$$

$$F_{cy} = \frac{G_{cy}(t_c + 2t_f)}{2t_f} \quad (9)$$

#### Interlaminar Stress

The core interlaminar shear stress is a constant and must be continuous at the facesheet/core interface. Interlaminar shear-stress distribution is independent of core properties. The maximum interlaminar shear stress occurs in the core, including the core/facesheet interface, and is an important aspect of interlaminar shear failure of sandwich structures. Empirical equations used for calculating the interlaminar strength ratio  $R$  are given by

$$R = \frac{\max(|S_{13}|, |S_{23}|)}{F_b} \quad (10)$$

#### Tsai–Wu Ply Failure Criterion

The well-known tensor polynomial theory of strength for anisotropic materials proposed by Tsai and Wu,<sup>19</sup> specialized to the case of an orthotropic lamina in a general state of plane stress, provides the strength ratio  $R$ , which is obtained as the positive square root of the quadratic:

$$AR^2 + BR - 1 = 0 \quad (11)$$

where  $A$  and  $B$  are given in terms of in-plane stresses and stress allowables as

$$A = \frac{S_x^2}{F_{tx}F_{cx}} + \frac{S_y^2}{F_{ty}F_{cy}} + \frac{S_{xy}^2}{F_{txy}^2} - \frac{S_x S_y}{(F_{tx}F_{cx}F_{ty}F_{cy})^{1/2}} \quad (12)$$

$$B = S_x[(1/F_{tx}) - (1/F_{cx})] + S_y[(1/F_{ty}) - (1/F_{cy})] \quad (13)$$

In this implementation, a MSC/NASTRAN analysis is performed by specifying the ply lay-up through PCOMP data cards and ply level material properties through MAT8 material cards. The ply stresses obtained from NASTRAN analyses are used to compute the laminate stresses. Classical lamination theory is used for computing the laminate level elastic properties from ply material properties. The ply and laminate stresses, in turn, are used to compute the ply and sandwich strength and local buckling margins of safety.

### III. Composite Sandwich Structure Optimization Problem

The composite sandwich structure optimization problem can be stated as a nonlinear programming problem in the following form:

Find the vector of design variables  $X$  that  
Minimizes:

$$F(X) \quad (\text{Objective function}) \quad (14)$$

subject to:

$$g_j(X) \leq 0, \quad j = 1 \text{ (Number of inequality constraints)} \quad (15)$$

$$x_i^l \leq x_i \leq x_i^u, \quad i = 1 \text{ (Number of design variables)} \quad (16)$$

For the composite sandwich structure optimization considered here, the design objective function is to minimize either the structural weight or the thermal distortion while satisfying the constraints imposed by Eqs. (15) and (16). The constraints include local instabilities (sandwich local buckling modes), stress requirements (ply level as well as laminate level), and frequency requirements imposed

to avoid dynamic coupling between two subsystem/component frequencies. Design variables used for optimization of composite sandwich structures include individual ply thickness, ply orientations, and honeycomb-core thickness. In addition, the tailoring of laminate stacking sequence can be achieved by identifying the location of a ply (having a particular thickness and orientation) in the laminate.

#### A. Augmented Function

The nonlinear optimization problem is posed to the GA as a minimization problem using the augmented function approach and penalty parameters. When any constraint is violated, a penalty is imposed and the augmented function of a design is calculated as

$$\phi = -W - \beta g_{\omega}(x_i) - \gamma \sum_{j=1}^{n_c} g_j(x_i) \quad (17)$$

$$\forall \quad g_{\omega}(x_i) < g_{\omega}^u(x_i), \quad g_j(x_i) > g_j^u(x_i)$$

where the penalty parameters  $\beta$  and  $\gamma$  are chosen as  $10^5$  and  $10^3$ , respectively. The number of violated constraints is denoted by  $n_c$ . When the constraints are satisfied, the augmented function is evaluated as

$$\phi = -W + \varepsilon g_{\omega}(x_i) + \varepsilon \sum_{k=1}^{n_s} \text{Mod}[g_k(x_i)] \quad (18)$$

$$\forall \quad g_{\omega}(x_i) \geq g_{\omega}^u(x_i), \quad g_k(x_i) \leq g_k^u(x_i)$$

where the total number of satisfied constraints is denoted as  $n_s$ . The penalty parameter  $\varepsilon$  is a small number (e.g., 0.0001) that allows designs having the same weight but differing margins of safety in the population of designs an incentive to survive for future generations.

### IV. GA and Representative Designs

The key to solving an optimization problem efficiently using GAs lies in the representation of the problem in terms of design variables. The design variables in the actual optimization problem are encoded in terms of alleles, which are interpreted by the GA. This encoding can vary depending on the problem being solved. The genetic algorithm is initiated by an initial population of designs created by seeding the initial population with random or appropriately chosen designs. Whether the designs are feasible or infeasible, they serve as initial starting points in the design space. The fitness value is assigned to every design based on its augmented function and the relative performance of each design in the ranked population (i.e., position of the design based on the evaluated augmented function with respect to the augmented function of the other designs in the population).

#### A. Selection Procedure

Two independent selection procedures are considered. The first procedure is a completely random selection in which the parent designs are selected on the basis of their hierarchy (evaluated as a function of the rank of the design based on the augmented function describing the fitness of the design). The second procedure ensures that the elitist design (best design) of the previous generation contributes to the next generation by reproducing with a random mate. The rest of the child designs are created by selecting random parents.

The selection process is biased so that high-performance designs have a higher probability of transmitting their features to the next generation. This is implemented by allocating shares of the roulette wheel to individuals in proportion to their fitness. The fitness assigned to the  $i$ th best design of a population of  $S$  designs is then equal to  $2(i - 1.0)/2S(2S - 1.0)$ . This procedure is called here the elitist selection so that the sum of all fitnesses is equal to 1. The roulette wheel is simulated by the segment  $[0, 1]$ , and a spin of the wheel is simulated by generating a random number between 0 and 1, which identifies the selected individual design.

#### B. GA Operators

The most creative step in the evolutionary process is creation of appropriate genetic operators for the particular problem addressed.

Past experience has indicated that algorithm efficiency for searching appropriate designs directly depends on the choice of operators applied to a particular problem. The genetic operators considered in the present work are the two-point crossover, mutation, permutation, and switching.

**Two-point crossover.** The crossover operator used in the present work is the traditional two-point crossover operator.<sup>6</sup> Two random points are identified for each of the selected parent designs, and the design information between the two identified points in each of the selected parents is exchanged to create new child designs. Crossover operation is applied with a probability chosen a priori, PCROSS, chosen herein as unity.

**Mutation.** This operator introduces small changes in the designs produced by crossover. The new design is randomly picked and endures mutation. Mutation is a stochastic operator, applied with a low probability (as compared to crossover), PMUTE, and it acts as a device that randomly introduces new genetic material into a design. It protects against complete loss of genetic material by changing a random allele in a string.

**Switching and permutation.** These operations are used with the composite laminate stacking sequence enumeration problem for generating new designs. The primary benefit of switching and permutation is the ability to produce new designs while maintaining constant the total number of occurrences of each variable in the design string. Operations like crossover and mutation change the number of occurrences of a certain allele in a laminate, and this may not always be permissible. In the stacking sequence enumeration problem, the crossover and mutation probabilities are put to zero (i.e., effectively, there is no crossover or mutation active) while the permutation and switching operations are applied with a finite probability.

**Permutation.** The permutation operation finds alternate designs using a single design in the population. Two random points are picked, and a new laminate is formed by shuffling the alleles between the two randomly chosen points.

**Switching.** The switch operator is applied with a probability of PSWITCH. Randomly chosen sections of the stacking sequence are exchanged to produce new designs. The switch operator aids in exploring the design space. Parameters such as starting locations for the first and second substrings and the length of the substring are determined in a random manner.

#### C. Combined Pool

The parents and children (after reproduction) form a combined set of designs (of size twice the actual population sizes) that is referred to herein as a combined pool. Upon calculation of the augmented function for the individual children designs, the parent and children designs are ranked based on their augmented function. The best of the combined pool set is selected to act as parents for the next generation. As part of selecting sets of new parents, it is possible to get duplicate designs that have the same augmented function values. In addition, there is a distinct possibility that individual designs (having the same augmented function) may or may not differ in terms of design variables in the string (since weight is affected by the discrete thicknesses rather than the ply orientations). To account for both possibilities and ensure that distinct strings are in the final parent population of designs, a duplication check is carried out. A design coefficient (DC) is evaluated as a function of the design variable value and its string location index for each design as follows:

$$DC_j = \sum_{k=1}^{n_l} x_k^j * k \quad \forall \quad j = 1, \dots, S \quad (19)$$

If the augmented function values of individual designs are identical, then the DC values for the designs are checked to ensure that the strings are different. The complete population of  $S$  designs is then filled using the best, selected, distinct individuals from the combined pool. If duplicate designs are present, then the designs are mutated to ensure that they will be different from previously occurring designs. The selected individuals will then be considered as parents for the next generation. The standard deviation for the combined pool population gives an indication of how the population tends either to

disperse or to concentrate in a small region of the design space. The scatter in the design fitness is measured by the standard deviation of the augmented objective function (SDAF) of the members of the population at any given generation and can be evaluated as

$$\text{SDAF} = \sqrt{\frac{\left[ S \sum_{k=1}^S \phi_k^2 - \left( \sum_{k=1}^S \phi_k \right)^2 \right]}{S(S-1)}} \quad (20)$$

The rate of convergence of the entire population as a whole is given by the slope of the SDAF curve when plotted as a function of the number of generations.

#### D. Least-Squares-Based Approximation Procedure for Genetic Search Algorithm

In this paper, a linear least-squares approximation technique is used with the GA to minimize the computational effort required for solving discrete ply lay-up and stacking sequence optimization problems. A cache, similar to the binary tree structure of Ref. 14, is used to store and retrieve the analysis data. The least-squares approximation for the design objective and constraint functions is used if a predefined number of design alternatives exist in the cache data structure within a specified distance from the design point to be approximated. These neighboring points, which are used for the approximation, are checked for linear independence or, in other words, they should be spatially differentiable. Since the linear approximation is in terms of either the design variables or a distance measure, the procedure investigated here is general and applicable to all analysis model responses.

To evaluate the design objective and constraint functions corresponding to a design vector, the cache data structure is first searched. If the analysis solution for the design vector of interest is available in the cache, the objective and constraint function values are retrieved. Otherwise, the cache is searched for neighboring designs of the design point of interest in the design variable space. In this work, a neighbor design is defined as a design point within 15 or 25% distance from the design point to be approximated. The Euclidean norm is used for computing the distances from cache points to design point. If sufficient neighbor design points are available and if they are linearly independent, then these linearly independent neighbor points are used with the following least-squares approximation to approximate the design point.

The function approximation is defined in the following form:

$$f(\mathbf{y}, \mathbf{a}) = a_0 + a_1 y_1 + a_2 y_2 + \dots + a_{ndv} y_{ndv} = a_0 + \sum_{k=1}^{ndv} a_k y_k \quad (21)$$

where  $\mathbf{y} = \{y_1, y_2, \dots, y_{ndv}\}$  are the translated design variables, such that the design point to be approximated is the center point. The translation is of the form  $\mathbf{y} = \mathbf{X} - \mathbf{X}_a$ , where  $\mathbf{X}_a$  is the design point to be approximated. To calculate the coefficient vector,  $\mathbf{a} = \{a_0, a_1, a_2, \dots, a_{ndv}\}$ , the least-squares procedure to minimize the error function  $\Phi$  is used.

Minimize:

$$\Phi = \sum_{k=1}^{\text{NNA}} [f_k - f(\mathbf{y}_k, \mathbf{a})]^2 \quad (22)$$

where NNA is the number of neighbor points used for approximation. The coefficient vector  $\{\mathbf{a}\}$  obtained from the least-squares solution using singular value decomposition is then used to approximate the function for the point of interest.

Alternatively, the least-squares approximation can also be performed in terms of a distance measure. In this case, the function approximation is defined as

$$f(d, \mathbf{a}) = a_0 + a_1 d \quad (23)$$

where  $d$  is the Euclidean distance from the center of the subspace formed by the neighbor points selected for the least-squares approximation to the design point to be approximated.

#### E. Implementation Details

In the context of SAT-OPT (Fig. 1), no sensitivity analysis is performed in this GA-based composite ply lay-up optimization procedure (Fig. 2).

For the implementation of GAs, each individual in the population representing a design alternative is represented by a finite string of numbers or digits. In the case of discrete ply angle optimization, the string corresponding to the laminate  $(90_2/\pm 45_2/90_2/0_2/\pm 45_2/0_2)_s$  is given by

$$\begin{array}{cccccccc} 90 & 90 & +45 & -45 & +45 & -45 & 90 & \\ 90 & 0 & 0 & +45 & -45 & +45 & -45 & 0 & 0 \end{array}$$

In the case of stacking sequence optimization, the string of digits corresponding to the same laminate is encoded as

$$3 \ 2 \ 2 \ 3 \ 1 \ 2 \ 2 \ 1$$

where the digits 1, 2, and 3 represent three stacks of  $0_2$ ,  $\pm 45$ , and  $90_2$ , respectively.

The design strings for the initial population are generated using a combination of prior or appropriately chosen designs. First, the initial population is evaluated to obtain the fitness values corresponding to each string. The constraints are taken into account as quadratic penalty terms in the augmented objective function. Based on the problem type and operator probabilities, genetic operators are applied to the design strings in the initial population to create the next generation of design strings.

The selection of design strings for successive generations is based on a combined population consisting of design strings before and after genetic operations. The top  $S$  strings are selected from this combined  $2S$  population without duplication. Crossover and mutation operators are used with discrete optimization problems, whereas switching and permutation operations are used with the stacking sequence problems. The generation evolution process is repeated until no improvement in the best design is obtained for five successive generations.

### V. Design Examples

The optimization methodology developed as part of SAT-OPT was used to design different composite sandwich structures. Design examples, including a satellite reflector, a satellite solar array substrate, and a composite panel, are presented here.

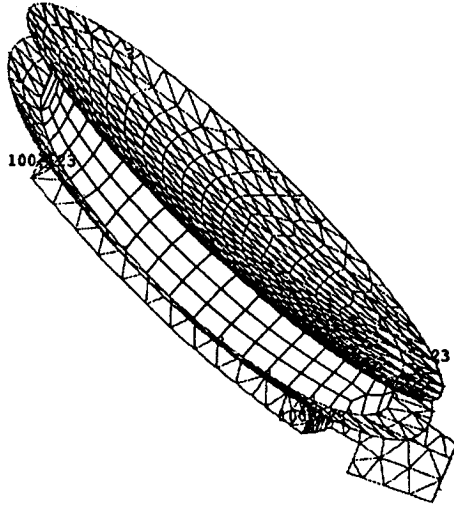
#### A. Satellite Reflector

The dual-surface parabolic reflector structure, shown in Fig. 4, is to be designed for minimum thermally induced surface distortions so as to improve the reflector pointing accuracy. The reflector consists of a front and rear dish, intercostal rings, and a backing structure. The rear dish and backing structure are made of composite facesheets and nonmetallic honeycomb core, and the intercostal rings and front reflector dish are made of composite facesheets and nonmetallic honeycomb core. The NASTRAN finite element model consists of over 1900 triangular and quadrilateral plate elements and a total of 6570 degrees of freedom. NASTRAN PCOMP cards were used to model the laminates because this permits each facesheet to have its own coefficient of thermal expansion (CTE), thus providing a capability to model any CTE mismatch between the top and bottom facesheets of the sandwich construction. The design load conditions include quasistatic transient loads, acoustic pressure loads, and on-orbit thermal cases, including worst-case hot and worst-case cold conditions. Thermal analyses are performed to determine the maximum and minimum temperature predictions and gradients corresponding to these cases and to provide inputs for NASTRAN thermal distortion and stress analyses.

The design optimization problem is formulated to minimize the thermally induced surface distortions at the center of the front reflector dish in the deployed configuration. Design constraints are imposed on structural weight, reflector frequency in stowed configurations, and local buckling failure margins of safety, ply, and sandwich strength. The design variable string for the GA includes seven variables that consist of composite sandwich facesheet and honeycomb core thickness corresponding to the front reflector dish

**Table 1** Satellite reflector thermal distortion optimization (responses normalized with respect to allowables)

Design variable string and normalized responses	Baseline design	Optimized design
$(t_f^F, t_c^F, t_c^R, t_f^I, t_c^I, t_f^B, t_c^B)$	(1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)	(1.375, 1.0, 1.0, 0.6875, 1.0, 1.0, 1.0)
Max thermal distortion @ center of front dish, in.	1.0	0.90
Structural weight, lb	1.0	1.06
Fundamental frequency, Hz	1.025	1.0
Ply failure margins of safety	>0.01	>0.01
Sandwich local buckling margins of safety	>0.01	>0.01

**Fig. 4** Satellite reflector structure.

$(t_f^F, t_c^F)$ , rear reflector dish  $(t_c^R)$ , intercostal ring  $(t_f^I, t_c^I)$ , and backing structure  $(t_f^B, t_c^B)$ .

The optimization solution resulted in a 10% reduction in the thermal distortions at the nodes at the center of the front dish, and the normalized values of the design objective and constraint responses are provided in Table 1.

#### B. Satellite Solar Array Substrate

The composite solar array substrate with doublers, shown in Fig. 5, is designed for weight, frequency, ply, and sandwich strength and for local buckling failure requirements. The substrate is made of composite faceskins and doublers and aluminum honeycomb core. The local use of doublers in heavily loaded portions of the substrate improves its quality by avoiding the use of heavier faceskins over the entire substrate. The NASTRAN finite element model of the substrate consists of 1056 quadrilateral plate elements and 6718 degrees of freedom. NASTRAN PCOMP and MAT8 cards are used to model the laminate lay-up and ply level material properties. The substrate is subjected to a quasistatic acoustic pressure load and on-orbit thermal soak loading conditions.

The substrate is made up of the following lay-up:

Substrate:  $(\theta_1/0_{\text{core}}/\theta_1)$  and  $t_1$  = facesheet thickness

Doubler:  $(\theta_1/\theta_2/-\theta_2/\theta_3/1-\theta_3/0_{\text{core}})_s$   
and  $t_2 = t_3$  = doubler ply thicknesses

The design optimization problem is to minimize the structural weight with constraints on the fundamental frequency, ply strength, and sandwich local buckling failure margins of safety to be positive. The design variables are the composite sandwich skin lay-up parameters, including ply thickness and ply orientation angles corresponding to the substrate and doubler regions.

The genetic algorithm was executed for a total of 20 generations. The designs obtained without approximations, with design variable approximations, and with distance approximations are compared in

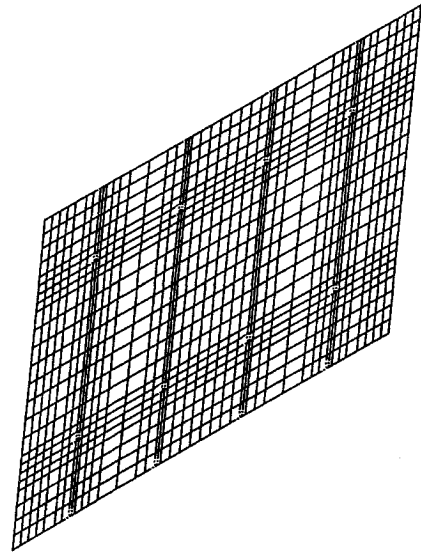
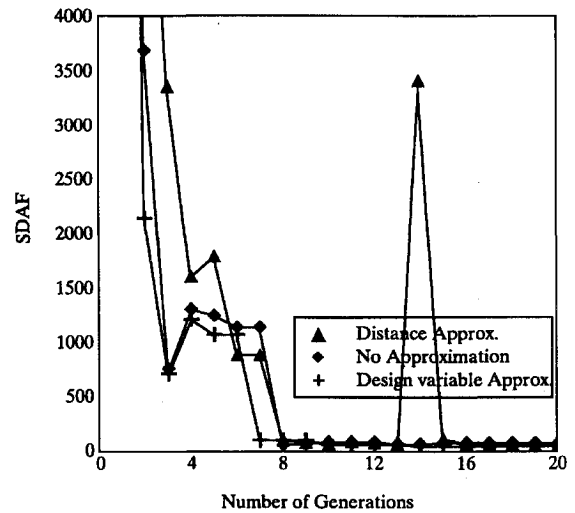
**Fig. 5** Satellite solar array substrate.**Fig. 6** Solar array substrate, SDAF curve.

Table 2. The actual values for ply thickness and orientation variables are normalized with respect to the maximum thickness and orientation angle of the baseline lay-up. In addition, the rate of convergence of the entire population as a whole is shown in Fig. 6, in the form of an SDAF vs generation curve. The spike seen in Fig. 6 depends on the SDAF value calculated based on the generation (current) and fitness of the individual members of the population in the generation. The effect of random operators contributes to the spike and, hence, it is a random event and not a physical event.

#### C. Composite Panel in Biaxial Compression

A simply supported, graphite-epoxy panel under biaxial compression loads of  $\lambda N_x$  and  $\lambda N_y$ , shown in Fig. 7, is considered. The panel geometry and loading are identical to those used in Ref. 10. The laminate is made up of  $N$  plies with orientations limited to  $\pm 45^\circ$ - and  $90^\circ$ -plies. The total percentage of a particular orientation (i.e., number of  $0^\circ$ -,  $90^\circ$ -, or  $45^\circ$ -plies has to remain the same) must remain constant. In this work, the panel was modeled using 20 noded, three-dimensional, solid finite elements. The laminated panel is modeled using multiple layers in each element and integrating each layer individually into the stiffness matrix.

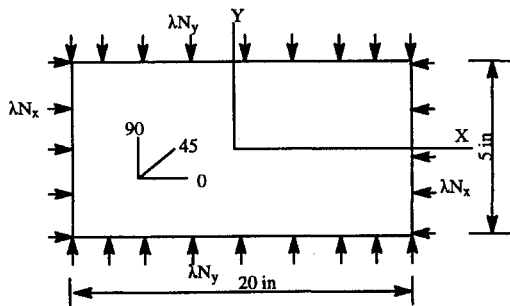
The orthotropic material properties used in the graphite-epoxy plate model are  $E_x = 19.2E + 6$  psi,  $E_y = 1.56E + 6$  psi,  $E_z = 1.56E + 6$  psi,  $G_{xy} = G_{xz} = 0.82E + 6$  psi,  $G_{yz} = 0.49E + 6$ ,  $\nu_{xy} = 0.24$ ,  $\nu_{yz} = 0.49$ ,  $\nu_{xz} = 0.24$ ,  $t_{\text{ply}} = 0.005$  in.<sup>11</sup>

**Table 2** Solar array substrate lay-up optimization with mutation probability of 0.5

Design variables	Frequency	Ply and sandwich failure margins of safety				No. of analyses in 20 generations
		Tsai-Wu	Intracell buckling	Core shear	Wrinkling	
Case 1 No approximations (normalized values)						
$t_1 = 1.00$						
$\theta_1 = 0.0$						
$t_2 = 0.42$	1.017	0.0833	0.3228	13.51	4.96	116
$t_3 = 0.42$						
$\theta_2 = 0.11$						
$\theta_3 = -0.11$						
Case 2 Distance approximations (normalized values)						
$t_1 = 1.00$						
$\theta_1 = 0.0$						
$t_2 = 0.625$	1.015	0.2483	0.3367	17.85	5.34	49
$t_3 = 0.21$						
$\theta_2 = 0.11$						
$\theta_3 = -0.11$						
Case 3 Design variable approximations (normalized values)						
$t_1 = 1.00$						
$\theta_1 = 1.0$						
$t_2 = 0.42$	1.017	0.0467	0.2446	8.44	1.97	62
$t_3 = 0.42$						
$\theta_2 = 0.11$						
$\theta_3 = -0.11$						
Solar array weight: same for all three cases						

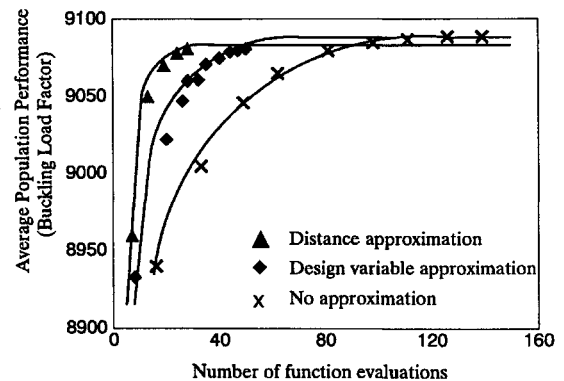
**Table 3** Composite panel stacking sequence optimization

Data	No approximation	Design variable-based approximation	Distance-based approximation
Design string	$(\pm 45_3/90_{12}/\pm 45_2/90_2)_s$	$(\pm 45_3/90_{12}/\pm 45_2/90_2)_s$	$(\pm 45_3/90_{10}/\pm 45/90_4/\pm 45)_s$
Critical buckling load factor	9093.05	9093.05	9092.64
Number of detailed analyses (10 generations)	172	67	57

**Fig. 7** Composite panel geometry and applied loading.

The design problem objective is to maximize the critical buckling load by varying the laminate stacking sequence. A load ratio of  $N_y/N_x = 0.5$  is considered. For the GA, a total of 10 generations, with a population size of 20 in each generation, is used. The initial population is generated randomly, using  $\pm 45^\circ$ - and  $90^\circ$ -deg plies.

The optimum stacking sequence for buckling critical design is obtained as (2 2 2 3 3 3 3 3 2 2 3), which corresponds to a  $(\pm 45_3/90_{12}/\pm 45_2/90_2)_s$  laminate. The buckling load factor corresponding to this design is 9093.5. The buckling critical design obtained here is different from the one reported in Ref. 11, which uses plate theory for the buckling solution. The optimal stacking sequence of Ref. 11 for critical buckling was  $(90_2/\pm 45_4/90_{12}/\pm 45)_s$ , with a critical buckling load factor of 9999.3. With the three-dimensional solid element formulation (using reduced integration order of 2), the buckling load factor for Ref. 11 design corresponds to 9077.8. The differences in the buckling solution are primarily attributed to the differences in the energy of the system

**Fig. 8** Convergence characteristics for composite panel.

modeled using three-dimensional solid elements and plate theory. Also, with the plate theory, as a result of balance symmetric assumptions, the terms  $D_{16}$  and  $D_{26}$  are not included in the stiffness contribution.

The genetic algorithm for this problem was run for a total of 10 generations. The designs obtained without approximations, with design variable approximations, and with distance approximations are compared in Table 3. The design variable approximation converges to the no approximation solution, whereas the distance approximation does not find the best lay-up. Although the design variable approximation is more accurate compared to the distance approximation, the requirements to approximate a design point in terms of design variables are much more rigorous to satisfy than the distance-based, least-squares approximation. Some performance aspects of the approximation procedure are shown in Figs. 8 and 9.

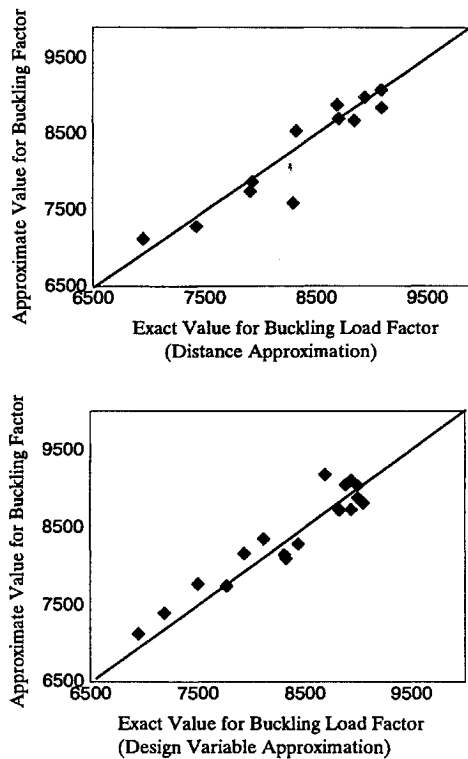


Fig. 9 Approximation quality for composite panel.

## VI. Summary

Of significant importance is the need to come up with lightweight, higher-quality, and more reliable satellite designs while reducing the overall design cycle time. Optimization technology in general and genetic algorithms in particular can be extremely useful for discrete tailoring of composite sandwich construction of satellite structures, providing for weight-efficient satellite designs. In addition, GAs provide for multiple local optimal designs, allowing the designer to pick an acceptable solution for fabrication and testing.

The sizing of composite sandwich structures is a very complex design task, considering the several modes of failure possible with such constructions. In this paper, failure modes induced by local instabilities associated with sandwich construction have been developed and used as design constraints in the optimization problem.

A linear least-squares approximation procedure is outlined to minimize the computational effort required for solving discrete composite ply lay-up optimization problems using GAs. Two forms of approximations, one with respect to the design variables and the other with respect to a distance measure, are investigated. Laminate lay-up parameters (facesheet thickness and orientation), honeycomb-core parameters, and laminate stacking sequence optimization problems are considered. Significant reductions in the number of detailed finite element analyses can be obtained by the use of a least-squares approximation based on either distance or design variables. In addition, the computational time required for this approximation is insignificant compared to a detailed finite element analysis. The quality of the approximation is sufficiently accurate to guide the selection of the design strings for successive generations.

For large-scale design optimization problems, the number of function evaluations required by this implementation of GAs is still considered to be high and more robust, and alternate approximation concepts are necessary to minimize the overall computational effort.

## References

- <sup>1</sup>Bruhn, E. F., *Analysis and Design of Flight Vehicle Structures*, Jacobs Publishing, Philadelphia, PA, 1973, Chap. C12.
- <sup>2</sup>Vinson, J. R., and Handel, P. I., "Optimal Stacking Sequences of Composite Faces for Various Sandwich Panels and Loads to Attain Minimum Weight," *Proceedings of the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics, and Materials Conference* (Williamsburg, VA), AIAA, Washington, DC, 1988, pp. 999–1004 (AIAA Paper 88-2334).
- <sup>3</sup>Bushnell, D., "Truss-Core Sandwich Design via PANDA 2," *Proceedings of the AIAA/ASME/ASCE/AHS 31st Structures, Structural Dynamics, and Materials Conference* (Long Beach, CA), AIAA, Washington, DC, 1990, pp. 1313–1332 (AIAA Paper 90-1070).
- <sup>4</sup>Schmit, L. A., and Mehrinfar, M., "Multilevel Optimum Design of Structures with Fibre-Composite Stiffened-Panel Components," *AIAA Journal*, Vol. 20, No. 2, 1982, pp. 138–147.
- <sup>5</sup>Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design: With Applications*, McGraw-Hill, New York, 1984.
- <sup>6</sup>Goldberg, D. E., *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley, Reading, MA, 1989.
- <sup>7</sup>Hajela, P., "Genetic Search—An Approach to the Nonconvex Optimization Problem," *AIAA Journal*, Vol. 26, No. 7, 1990, pp. 1205–1210.
- <sup>8</sup>Callahan, K. J., and Weeks, G. E., "Optimum Design of Composite Laminates Using Genetic Algorithms," *Composites Engineering*, Vol. 2, No. 3, 1992, pp. 149–160.
- <sup>9</sup>Ball, N. R., Sargent, P. M., and Ige, D. O., "Genetic Algorithm Representation for Laminate Layouts," *Artificial Intelligence in Engineering*, Vol. 8, No. 2, 1993, pp. 99–108.
- <sup>10</sup>Le Riche, R., and Haftka, R. T., "Optimization of Laminate Stacking Sequence for Buckling Load Maximization by Genetic Algorithm Approach," *AIAA Journal*, Vol. 31, No. 5, 1993, pp. 951–956.
- <sup>11</sup>Nagendra, S., Haftka, R. T., and Gurdal, Z., "Optimization of Laminate Stacking Sequence with Stability and Strain Constraints," *AIAA Journal*, Vol. 30, No. 8, 1992, pp. 2132–2137.
- <sup>12</sup>Nagendra, S., Jestin, D., Gurdal, Z., Haftka, R. T., and Watson, L. T., "Improved Genetic Algorithm for the Design of Stiffened Composite Panels," *Proceedings of the 10th SIAM International Conference of Mathematical Programming* (Detroit, MI), Society for Industrial and Applied Mathematics, 1994.
- <sup>13</sup>Schmit, L. A., and Miura, H., "Approximation Concepts for Structural Synthesis," NASA CR-2552, March 1976.
- <sup>14</sup>Kogiso, N., Watson, L. T., Gurdal, Z., and Haftka, R. T., "Genetic Algorithms with Local Improvement for Composite Laminate Design," *Structural Optimization* (to be published).
- <sup>15</sup>Acikgoz, M., and Kodiyalam, S., "An Approximation Procedure for Genetic Search Strategy Based Composites Ply Layout Optimization," *Proceedings of the AIAA/NASA/USAF 5th Symposium on Multidisciplinary Analysis and Optimization* (Panama City, FL), AIAA, Washington, DC, 1994, pp. 1006–1014 (AIAA Paper 94-4362).
- <sup>16</sup>Kodiyalam, S., Graichen, C., Connell, I. J., and Finnigan, P. M., "Object-Oriented, Optimization-Based Design of Satellite Structures," *Journal of Spacecraft and Rockets*, Vol. 31, No. 2, 1994, pp. 312–318.
- <sup>17</sup>Vanderplaats, G. N., *ADS User's Manual*, VMA Engineering, Colorado Springs, CO, 1988.
- <sup>18</sup>Kodiyalam, S., "Transient Flight Loads Estimation for Spacecraft Structural Design," *Computing Systems in Engineering*, Vol. 5, No. 3, 1994, pp. 275–283.
- <sup>19</sup>Tsai, S. W., and Wu, E. M., "A General Theory of Strength for Anisotropic Materials," *Journal of Composite Materials*, Vol. 5, 1971, pp. 58–80.